

# Magnetic-field-induced Stoner transition in a dilute quantum Hall system

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In a recent paper [Phys.Rev.B.**84**, 161307 (2011)], experimental data on spin splitting in the integer quantum Hall effect has been reported in a high mobility dilute 2D electron gas with electron density as low as  $0.2 \times 10^{11} \text{ cm}^{-2}$ . In this work, we show that an excellent *quantitative* description of these data can be obtained within the model of the magnetic-field-induced Stoner transition in the quantum Hall regime. This provides a powerful tool to probe the non-trivial density dependance of electron-electron interactions in the dilute regime of the 2D electron gas.

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The integer quantum Hall effect (IQHE) observed at low temperature in a 2-dimensional electron gas (2DEG) subjected to a perpendicular magnetic field can partly be interpreted using a single particle picture. The Landau quantization gives rise to cyclotron gaps in the electronic density of states, and the longitudinal resistance goes to zero each time the number of occupied Landau level (the filling factor  $\nu$ ) is an even integer. The odd, or “spin-resolved” integer quantum Hall effect, characterized by the occurrence of zero resistance states at *odd* filling factors is, however, essentially a *many-body* phenomenon. Following the seminal theoretical work of Fogler and Shklovskii,<sup>1</sup> we have shown that the lifting of the electron spin degeneracy in the integer quantum Hall effect at high filling factors should be interpreted as a magnetic-field-induced Stoner transition.<sup>2</sup> A simple model with no free parameters correctly predicts the magnetic field required to observe spin splitting in the longitudinal resistance, confirming that the odd IQHE is a result of a competition between the disorder induced energy cost of flipping spins and the exchange energy gain associated with the polarized state. This can be thought of as a Stoner transition, since the only role played by the magnetic field is to modify the density of states at the Fermi energy. More generally, the absence of ferromagnetism in a 2DEG at zero magnetic field is related to the electron-electron correlations which prevent the Hartree-Fock spin susceptibility from diverging.

Recently, interesting new experimental data on the emergence of spin splitting in the integer quantum Hall effect has been reported<sup>3</sup> in a GaAs 2DEG where the density  $n_s$  can be tuned down to a value as low as  $0.2 \times 10^{11} \text{ cm}^{-2}$ , while preserving a high mobility ( $\mu > 2 \times 10^6 \text{ cm}^2/\text{V s}$ ). This data extends the study of spin splitting to a lower electron density range ( $n_s \ll 10^{11} \text{ cm}^{-2}$ ), where the role of many-body effects is enhanced and more complex. The aim of this brief report is to show that an excellent *quantitative* description of these data can be obtained within the Stoner model.<sup>2</sup>

To describe the appearance of spin splitting quantitatively, the Fogler and Shklovskii “half-resolved spin splitting” criteria has been retained consistently with previous work. This corresponds to the condition  $\delta\nu = 0.5$ ,

where  $\delta\nu$  is the filling factor difference between two consecutive resistance maxima in  $R_{xx}(B)$  related to spin up and down sub-levels associated with a given Landau level. According to the Stoner model, the critical odd filling factor  $\nu_c$  (magnetic field  $B_c = n_s h / e \nu_c$ ) for a half-split Landau level *i.e.* with  $\delta\nu = 0.5$  must verify in the zero temperature limit the following set of self-consistent equations:<sup>4</sup> for the total spin gap  $\Delta_s$ ,

$$\Delta_s = |g^*| \mu_B B + X \frac{1}{2} (e B_c / h), \quad (1)$$

and for the spin polarization within the Landau level,

$$\delta\nu = \frac{1}{2} = \frac{1}{e B_c / h} \int_{-\infty}^0 \left[ D \left( E + \frac{\Delta_s}{2} \right) - D \left( E - \frac{\Delta_s}{2} \right) \right] dE, \quad (2)$$

where  $X$  is the exchange energy between two spins, essentially depending only on the electron density, and  $D(x) = (1/\Gamma\sqrt{\pi}) \exp(-x^2/\Gamma^2)$  is the normalized density of states for a Gaussian broadened spin Landau level of full-width at half-maximum  $2\sqrt{\ln(2)}\Gamma$ .

In figure 1, we plot the critical Landau level index,  $N_c = (\nu_c - 1)/2$ , at which the spin splitting is resolved for a given electron density  $n_s$ . Note that in this plot the magnetic field is not constant but equal to  $n_s h / e \nu_c$ . The red dots are the experimental results of Ref. [3] (figure 3). It can be seen that the critical filling factor  $\nu_c$  shifts down as the density is reduced. This is because for a given strength of the exchange interaction and a given amount of disorder, there is a critical magnetic field which is required for the spin splitting to be resolved.<sup>2</sup> The open stars are the results obtained using Eqs. 1 and 2. The density dependant exchange parameters  $X(n_s)$  has been calculated with the same approach as in Ref. [2], by using a theoretical spin susceptibility including finite thickness corrections<sup>5</sup> which are non-negligible for the wide 2DEG studied in Ref. [3]. The only remaining adjustable parameter, the Landau level broadening  $\Gamma$  is found to give the best fit for  $\Gamma = 0.457 \text{ K}$  ( $\sim 0.04 \text{ meV}$ ).

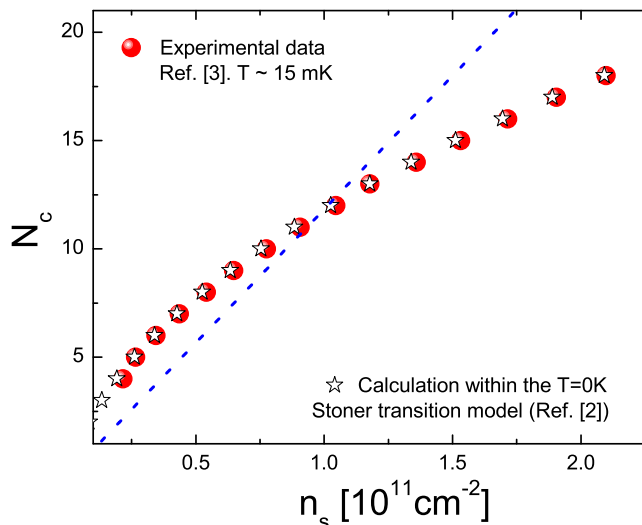


FIG. 1: (Color online) Critical Landau level index  $N_c$  at which the spin splitting is resolved (see text) as a function of the electron density  $n_s$ . Experimental results of Ref. [3] (red dots) and calculations within the Stoner transition model for spin splitting (see text) (open stars). The dotted line is the result for a fixed (density independent) value of the exchange parameter  $X$ .

An excellent agreement between experiment and theory is obtained, showing crucially that the Stoner transition describes spin splitting accurately down to the dilute regime investigated here.<sup>7</sup> The effect of a non-zero Zeeman energy is included and shifts the phase transition to higher Landau level index. The difference between the  $g^* = -0.44$  and  $g^* = 0$  results (not shown for clarity) is at most  $\sim 10\%$ , confirming the secondary role of the Zeeman energy in GaAs in the perpendicular magnetic field configuration.<sup>4,6</sup> Nevertheless, we stress that the Zeeman energy has to be included to obtain such a good qualitative and quantitative agreement with experimental data.

The resulting value of the Landau level broadening,  $\Gamma = 0.457$  K, is reasonable given the high quality of the 2DEG of Ref. [3]. We have checked that the density dependence of the transport lifetime (which can be estimated from figure 1 in of Ref. [3]) has almost no influence on our results. The corresponding Landau level “transport broadening” is at most  $\Gamma_{tr} = 0.04$  K, which is still an order of magnitude below the total Landau broadening used here. This is related to the scattering being essentially long-range in such high mobility samples. One should mention that, as exchange interaction and disorder play an opposite role, any error made in estimating the exchange energy will have an influence on the resulting value of  $\Gamma$ . Ideally, these parameters should be independently determined by complimentary measurements on a given sample, as detailed in Ref. [2].

In Ref. [3], the experimental data are compared with the Fogler and Shklovskii (FS) theoretical result obtained using Eq.4c of Ref. [1] with the parameters of the inves-

tigated sample. Theory gives a good qualitative description of the density dependence of  $N_c$  but lies  $\sim 30\%$  below the experimental data. There are several possible reasons for this discrepancy. We recall the main differences between our model and the theory of FS. Firstly, the Zeeman energy is not included in Eq.4c of Ref. [1], which can lead as we mentioned above to a difference of  $\sim 10\%$ . Secondly, the exchange energy in our work is taken from calculations of the spin susceptibility showing a good agreement with experimental data for electron densities much lower than  $10^{11} \text{ cm}^{-2}$  (see figure 3 in Ref. [5]). In the theory of FS, it is stressed that the result is obtained within the approximation  $k_F a_B > 1$ , where  $k_F$  is the Fermi wave factor and  $a_B$  is the effective Bohr radius of the 2DEG in GaAs, which corresponds to an electron density  $n_s > 1.8 \times 10^{11} \text{ cm}^{-2}$ . Finally, disorder is taken into account in our approach with an adjustable parameter ( $\Gamma$ ), whereas the disorder contribution in Ref. [3] is calculated from Eq.4c of Ref. [1] with experimental parameters extracted for the investigated sample.

We now focus on the behavior of  $N_c$  in the low density region ( $n_s < 10^{11} \text{ cm}^{-2}$ ). Interestingly, In this region,  $N_c(n_s)$  exhibits a pronounced sub-linearity. This behavior is very well caught by the model provided the density dependence of the exchange interaction is taken into account. To illustrate this we also plot the result obtained using Eqs. 1 and 2 with a fixed value of  $X$ , taken to be its value at  $n_s = 1 \times 10^{11} \text{ cm}^{-2}$  (dotted line). No concave trend is predicted in this case at low densities. This can be related extremely simply to the prediction of the critical magnetic field for the *appearance* of spin splitting in Ref. [2], corresponding to the case  $\delta\nu = 0$ :

$$B_{ss} = (n_s h) / (e \nu_{ss}) = \frac{\hbar \Gamma \sqrt{\pi}}{e} \frac{1}{X(n_s)}. \quad (3)$$

The critical filling factor  $\nu_{ss}$  for the appearance of spin splitting should be a linear function of the density  $n_s$  with a slope  $\propto \frac{X}{\Gamma}$ , unless the exchange parameters  $X$  or the disorder broadening  $\Gamma$  depends on density. The experimental behavior at low density can thus be attributed to the theoretically predicted increase in exchange energy as the density is reduced (see e.g. Ref. [5,9]) giving rise to the concavity of  $N_c(n_s)$ . The low density behavior of spin splitting in the integer quantum Hall regime combined with the Stoner transition approach thus provides a powerful tool to probe the non-trivial density dependance of electron-electron interactions in the dilute regime.

To conclude, we have shown that the magnetic-field-induced Stoner transition approach previously used to describe the spin-resolved quantum Hall effect can be extended to electron densities as low as  $0.2 \times 10^{11} \text{ cm}^{-2}$ . In this dilute regime, the so-called  $r_s$  parameter corresponding to the ratio of the Coulomb energy to the kinetic energy reaches  $r_s \sim 4.3$ . In this situation electron-electron correlations effects become important and can be experimentally probed by studying the low magnetic

field spin splitting in the quantum Hall regime.

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